Grounded fixpoints

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Operator-based semantics

- Many logics: semantics defined as (some sort of) fixpoints of a semantic operator
  - Logic programming
  - Autoepistemic logic
  - Default logic
  - Dung's argumentation frameworks
  - Abstract dialectical frameworks

Operator-based semantics

- Many logics: semantics defined as (some sort of) fixpoints of a semantic operator
  - Logic programming
  - Autoepistemic logic
  - Default logic
  - Dung's argumentation frameworks
  - Abstract dialectical frameworks
- Lots of similarities, e.g.,
  - Monotone operator: least fixpoint
  - Non-monotone operator: harder to find "good fixpoints"

Approximation Fixpoint Theory

- Denecker, Marek and Truszczyński (2000) (DMT)
- Algebraical theory
- Defines different types of fixpoints of lattice operators
  - Supported fixpoints
  - (Partial) stable fixpoints
  - Kripke-Kleene fixpoint
  - Well-founded fixpoint
- Captures semantics of many logical formalisms
  - Logic programming
  - Autoepistemic logic
  - Default logic
  - Dung's argumentation frameworks (Strass, 2013)
  - Abstract dialectical frameworks (Strass, 2013)

This paper

- Study phenomenon that occurs in the above domains
- Groundedness $\leftrightarrow$ self-supporting

Logic Programming

- Completion semantics (Clark, 1978)

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$$\{p \leftarrow p\} : \text{ self-supporting model } \{p\}$$

Logic Programming

- Completion semantics (Clark, 1978)
- Perfect model semantics (Przymusinski, 1988)
- Well-founded semantics (Van Gelder et. al., 1988)
- Stable semantics (Gelfond and Lifschitz, 1988)

Autoepistemic Logic (AEL)

- Expansion semantics (Moore, 1985)

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$\{Kp \Rightarrow p\}$: self-supporting model in which p is known

Autoepistemic Logic (AEL)

- Expansion semantics (Moore, 1985)
- "Honest theories" (Halpern and Moses, 1985)
- Moderately grounded expansions (Konolige, 1988)
- Strongly grounded expansions (Konolige, 1988)
- Stable semantics (DMT, 2000)
- Well-founded semantics (DMT, 2000)

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Grounded fixpoints

- What is the problem with logic programs as $\{p \leftarrow p\}$ or AEL theories such as $\{Kp \Rightarrow p\}$?
- Algebraical study (fixpoint theory)
- Application to logic programming, autoepistemic logic, default logic, Dung's argumentation frameworks and abstract dialectical frameworks

Grounded fixpoints

- Complete lattice $\langle L, \leq \rangle$: every set $S \subseteq L$ has a least upper bound $\bigvee S$ and a greatest lower bound $\bigwedge S$
- Operator $O : L \to L$

Definition (Grounded)

We call $x \in L$ *grounded* for O if for each $v \in L$ such that $O(x \wedge v) \leq v$, it holds that $x \leq v$.

Grounded fixpoints: intuitively

- Intuition, if $L = 2^F$, $\leq = \subseteq$
- Then: $\bigwedge = \bigcap$ and $\bigvee = \bigcup$

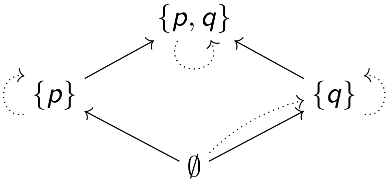
Definition (Grounded)

We call $x \in L$ *grounded* for O if for each $v \in L$ such that $O(x \wedge v) \leq v$, it holds that $x \leq v$.

x is grounded for O if it only contains facts that are sanctioned by O: whenever we remove facts from x, at least one of them is rederived.

Groundedness: Example

$$\left\{\begin{array}{l} p \leftarrow p. \\ q \leftarrow \neg p \vee q. \end{array}\right\}$$



Properties of grounded fixpoints

Proposition

All grounded fixpoints of O are minimal fixpoints of O.

Proposition

If $\mathcal{P}$ is a logic program, then grounded fixpoints of $T_{\mathcal{P}}$ are minimal supported models of $\mathcal{P}$.

Properties of grounded fixpoints

Proposition

A monotone operator has exactly one grounded fixpoint, namely its least fixpoint.

Proposition

If $\mathcal{P}$ is a positive logic program, then $T_{\mathcal{P}}$ has a unique grounded fixpoint, namely the least supported model of $\mathcal{P}$.

Properties of grounded fixpoints

Proposition

All A-stable fixpoints of O are grounded fixpoints of O.

Proposition

All stable models of a logic program $\mathcal{P}$ are grounded fixpoints of $T_{\mathcal{P}}$.

Properties of grounded fixpoints

Proposition

The well-founded fixpoint of a symmetric approximator A of O approximates all grounded fixpoints of O.

Proposition

The well-founded model of a logic program $\mathcal{P}$ is less precise than every grounded fixpoint of $T_{\mathcal{P}}$.

Properties of grounded fixpoints

Proposition

If the well-founded fixpoint of a symmetric approximator A of O is exact, then it is the unique grounded fixpoint of O.

Proposition

If the well-founded model of a logic program $\mathcal{P}$ is two-valued, then it is the unique grounded fixpoint of $T_{\mathcal{P}}$.

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Grounded fixpoints in logic programming

- Study of existing semantics: stable and two-valued well-founded semantics are "grounded"
- Closely related to unfounded sets
- Grounded fixpoints induce a new semantics
    - Two-valued
    - Purely algebraical
    - Simple
    - Easily extensible

Grounded fixpoints in autoepistemic logic

- Superior formalisation of Konolige's intuitions regarding "groundedness" (Konolige, 1988)
- The closest he got was "strong groundedness", a syntactical criterion that depends on which rewriting to a normal form is used
- All strongly grounded expansions are *grounded fixpoints* and all *grounded fixpoints* are moderately grounded

Other applications of grounded fixpoints

- Default logic
- Dung's argumentation frameworks
- Abstract dialectical frameworks

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Results

- Abstract algebraical definition of groundedness
- Studied relation with other types of fixpoints from AFT
- Corresponds to intuitions in many different domains
- Study of existing semantics
- New induced semantics with attractive properties
- Studied complexity

Current and Future Work

- Grounded fixpoints (AAAI'15)
- Grounded fixpoints and their applications in knowledge representation (submitted)
- Partial grounded fixpoints (submitted)

(Bogaerts, Vennekens and Denecker)