Partial Grounded Fixpoints

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Context: Grounded fixpoints

- “Grounded fixpoints” (AAAI, 2015)
- “Grounded fixpoints and their applications in knowledge representation” (AIJ, 2015)
- “Groundedness in logics with a fixpoint semantics” (PhD thesis, 2015)

Groundedness

- Different research domains:
  - Logic programming,
  - Autoepistemic logic,
  - Default logic,
  - Dung's argumentation frameworks,
  - Abstract dialectical frameworks.
- Similar intuitions: facts (or models)
  - are *grounded*,
  - are supported by *cycle-free* arguments,
  - are not *unfounded*,
  - can be built *from the ground up*.

Logic Programming

- Completion semantics (Clark, 1978) ungrounded models
- Perfect model semantics (Przymusinski, 1988)
- Well-founded semantics (Van Gelder et. al., 1988)
- Stable semantics (Gelfond and Lifschitz, 1988)

$$\{p \leftarrow p\}: \text{ self-supporting model } \{p\}$$

Autoepistemic Logic (AEL)

- Expansion semantics (Moore, 1985) ungrounded expansions
- "Honest theories" (Halpern and Moses, 1985)
- Moderately grounded expansions (Konolige, 1988)
- Strongly grounded expansions (Konolige, 1988)
- Constructive tightly grounded autoepistemic reasoning (Niemelä, 1991)
- Stable semantics (Denecker, Marek and Truszczyński, 2000)
- Well-founded semantics (Denecker, Marek and Truszczyński, 2000)

$\{Kp \Rightarrow p\}$: self-supporting expansions in which p is known

Grounded fixpoints

- What is the problem with logic programs as $\{p \leftarrow p\}$ or AEL theories such as $\{Kp \Rightarrow p\}$?
- Algebraical study (fixpoint theory)
- Application to logic programming, autoepistemic logic, default logic, Dung's argumentation frameworks and abstract dialectical frameworks

Grounded fixpoints

Given

- *Complete lattice $\langle L, \leq \rangle$: every set $S \subseteq L$ has a least upper bound $\bigvee S$ and a greatest lower bound $\bigwedge S$*
- *Operator $O : L \to L$*

Definition (Grounded)

We call $x \in L$ *grounded* for O if for each $v \in L$ such that $O(x \wedge v) \leq v$, it holds that $x \leq v$.

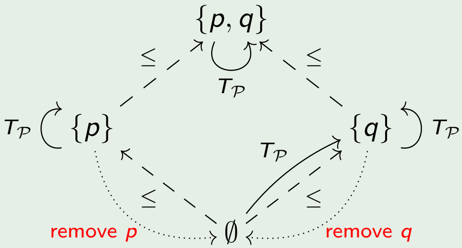
Intuition

- $L = 2^F, \leq = \subseteq$
- *In this case: x is grounded for O if it only contains facts that are sanctioned by O: whenever we remove facts from x, at least one of them is rederived.*

Groundedness

Example

$$\mathcal{P} = \left\{ \begin{array}{l} p \leftarrow p. \\ q \leftarrow \neg p \lor q. \end{array} \right\}$$



Grounded fixpoints in logic programming

- Study of existing semantics: stable and two-valued well-founded semantics are "grounded"
- Closely related to unfounded sets
- Grounded fixpoints induce a new semantics
  - Two-valued
  - Purely algebraical
  - Simple
  - Easily extensible
- Study complexity

Other applications of grounded fixpoints

- Autoepistemic logic
- Default logic
- Dung's argumentation frameworks
- Abstract dialectical frameworks

Results

- Abstract algebraical definition of groundedness
- Studied relation with other types of fixpoints from AFT
- Corresponds to intuitions in many different domains
- Study of existing semantics
- New induced semantics with attractive properties

This paper: Motivation

- Grounded fixpoints: limited to two-valued setting
- Sometimes partial fixpoints (three-valued models) are interesting:
  - Central objects of study
  - More general
  - As an analysis tool (e.g., in debugging)

Partial Grounded Fixpoints

Definition

Let A be an approximator of O. A point $(x, y) \in L^c$ is *A-grounded* if for every $v \in L$ with $A(x \wedge v, y \wedge v)_2 \leq v$, also $y \leq v$.

Properties of partial grounded fixpoints

Proposition

If A is a symmetric approximator of O, then x is grounded for O if and only if (x, x) is A-grounded.

Proposition

All consistent (partial) A-stable fixpoints are A-grounded.

Theorem

The well-founded fixpoint of a symmetric approximator A of O is the least precise A-grounded fixpoint.

Proposition

If A and B are approximators of O and $A \leq_p B$, then all consistent B-grounded points are also A-grounded.

Application (Logic Programming)

- Study semantical relationship with well-founded and stable models
- Closely related to unfounded sets
- Study complexity

Conclusion

Contributions:

- Abstract algebraical definition of groundedness (of bilattice points)
- Studied relation with other types of fixpoints from AFT
- Generalised properties of grounded fixpoints
- Corresponds to existing intuitions

Read more:

- "Grounded fixpoints" (AAAI, 2015)
- "Grounded fixpoints and their applications in knowledge representation" (AIJ, 2015)
- "Groundedness in logics with a fixpoint semantics" (PhD thesis, 2015)
- "Partial grounded fixpoints" (IJCAI, 2015)