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# Consistency in Justification Theory

**NMR 2018**

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October 29, 2018

# 0 Outline

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## 1 Aims of justification theory

- Unifying framework for describing various semantics of various logics
  - Logic programs
  - Abstract argumentation
  - Inductive definitions
  - Nested definitions

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- Unifying framework for describing various semantics of various logics
  - Logic programs
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  - Inductive definitions
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- By capturing the underlying types of *constructions* = *justifications*
- Give rise to new semantics
- Provides ways for seamless integration of various expressive language constructs
  - Aggregates into logic programs

# 1 Aims of justification theory: computational aspects

Justifications as datastructures in solvers

- Compute unfounded sets in ASP solvers (De Cat, Gebser)
- Check for relevance in complete search algorithms (Jansen)
- Lazy grounding (De Cat, Bogaerts)

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## 2 What is justification theory (intuitively)

- Theory (or program) in a logic induces a semantic structure: *justification frame*: a space of facts, a set of elementary construction steps
- *justification* := graph of facts constructed by elementary construction steps
  - that shows a type of construction of its facts
  - that embodies a potential reason why its facts are true
- A justification is a *good* construction if all its branches are *good*
  - Branch evaluation $\mathcal{B}(x_0 \to x_1 \to \dots)$ is a fact, true or false
- Different notions of branch evaluations
  $\Rightarrow$ different sorts of construction
  $\Rightarrow$ different types of semantics

## 2 Justification status of a fact in an interpretation $\mathcal{I}$

- Interpretations, possibly 4-valued $\mathbf{t}$ (true), $\mathbf{f}$ (false), $\mathbf{u}$ (unknown) and $\mathbf{i}$ (inconsistent)
  - Truth order $\mathbf{f} \leq_t \mathbf{u} \leq_t \mathbf{t}$, $\mathbf{f} \leq_t \mathbf{i} \leq_t \mathbf{t}$
- The support value of $J$ for a fact $x$ in $\mathcal{I}$ is the value of the worst branch $x \to x_1 \to \ldots$ in $J$ under the branch evaluation $\mathcal{B}$.

**Definition**

The *supported value of a fact* $x$ *in* $\mathcal{I}$ is the support value of the "best" justification for $x$.

Notation: $\mathrm{SV}(\mathcal{I}, x)$

# 2 A justification frame: example

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  - Facts $\mathrm{Edge}(v, w)$ and $\sim\mathrm{Edge}(v, w)$ for $v, w \in V$
  - Facts $\mathrm{Reach}(v)$ and $\sim\mathrm{Reach}(v)$ for $v \in V$

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- Rules of the justification frame:
  - $\mathrm{Reach}(a) \leftarrow \mathbf{t}$
  - $\mathrm{Reach}(v) \leftarrow \mathrm{Reach}(x), \mathrm{Edge}(x, v)$ for $v, x \in V$
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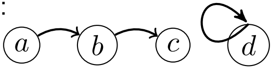
- Elements $\mathrm{Edge}(v, w)$ correspond to parameters
- A specific graph corresponds to an interpretation $\mathcal{I}$

Let the graph $(V, E)$ be:

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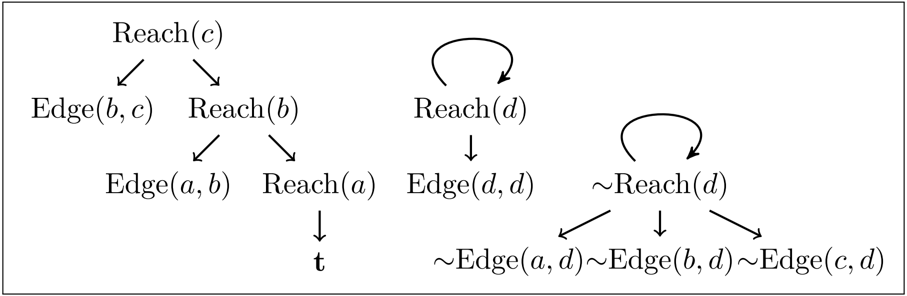
Part of a justification:



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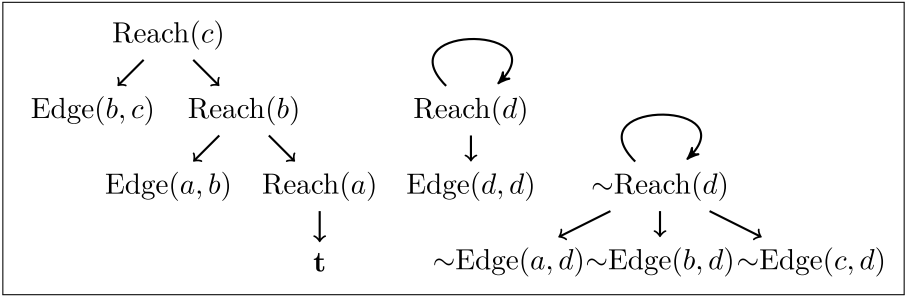


Under the suitable branch evaluation $\mathcal{B}$:

- finite branches evaluate to their leaf
- infinite branches of positive facts: evaluate to $\mathbf{f}$
  $\mathrm{Reach}(d) \rightarrow \mathrm{Reach}(d) \rightarrow \cdots$ is mapped to $\mathbf{f}$
- infinite branches of negative facts evaluate to $\mathbf{t}$
  $\sim\mathrm{Reach}(d) \rightarrow \sim\mathrm{Reach}(d) \rightarrow \cdots$ is mapped to $\mathbf{t}$

$\mathrm{SV}(\mathcal{I}, \mathrm{Reach}(c)) = \mathbf{t}, \mathrm{SV}(\mathcal{I}, \sim\mathrm{Reach}(d)) = \mathbf{t}$

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- Denecker et al. (2015) give characterisations of admissable, stable, preferred, complete and grounded sets in terms fixed points of an operator associated with the supported value

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- All four can be captured in justification theory with various branch evaluations

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- These values should not conflict, otherwise the semantics is defect
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  - ($\sim \mathbf{f} = \mathbf{t}$, $\sim \mathbf{t} = \mathbf{f}$, $\sim \mathbf{u} = \mathbf{u}$ and $\sim \mathbf{i} = \mathbf{i}$)
  - If $\mathrm{SV}(\mathcal{I}, \sim x) = \sim \mathrm{SV}(\mathcal{I}, x)$ for all $x$, then $\mathrm{SV}(\mathcal{I}, \cdot)$ is also an interpretation

## 3 Resolving conflicting status

**Theorem**

*If the rules for $x$ and $\sim x$ are "complementary", then*

$$\mathrm{SV}(\mathcal{I}, x) \leq_t \sim\mathrm{SV}(\mathcal{I}, \sim x)$$

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**Example of complementary**

Abstract Argumentation Frame: $A = \{a, b, c\}$ with
$R = \{(a, b), (c, b), (c, a), (a, c)\}$

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- Rules $\{\sim b \leftarrow a \qquad \sim b \leftarrow c \qquad \sim a \leftarrow c \qquad \sim c \leftarrow a\}$
- But also rules $\{b \leftarrow \sim a, \sim c \qquad a \leftarrow \sim c \qquad c \leftarrow \sim a\}$

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**Theorem**

*For the branch evaluations capturing Clarks completion, Kripke-Kleene, stable and well-founded semantics and "complementary" rules we have*

$$\mathrm{SV}(\mathcal{I}, x) = \sim\mathrm{SV}(\mathcal{I}, \sim x)$$

- Proof uses heavy machinery and clever "pasting" of justifications

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- Can we find a general property of branch evaluations so that $\mathrm{SV}(\mathcal{I}, x) = {\sim}\mathrm{SV}(\mathcal{I}, {\sim}x)$?
- What other formalisms can be expressed in justification theory?

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    - Logic programming semantics
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  - Used in computational tools
- Consistency result for particular semantics
  - Clarks completion
  - Kripke-Kleene
  - Stable
  - Well-founded