# Towards a Lower Bound Founded Fixpoint Semantics: Working Abstract

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A distinguishing feature of answer set programming (ASP; [2]) is that all atoms belonging to a stable model must be founded. This can be made precise by means of the constructive logic of here-and-there (HT; [9]), whose equilibrium models correspond to stable models [10]. Foundedness can be regarded by assigning the smallest Boolean truth value that can be proven, where false is smaller than true. In recent work, this idea was extended to constructs over non-Boolean domains, among others in the context of HT [4], resulting in the so-called lower bound-founded logic of here-and-there ($\mathrm{HT}_{\mathrm{LB}}$). Let us illustrate this logic.

*Example 1.* Imagine, we have a car and know about its maximal range it can go and its revolutions per minute (rpm). Then, we model the dependency of increasing rpm to decreasing range by the following set of $\mathrm{HT}_{\mathrm{LB}}$ rules

$$\mathrm{rpm} \geq 2000. \qquad \mathrm{range} \geq 100. \qquad \mathrm{range} \geq 200 \leftarrow \mathrm{rpm} < 4000.$$

In $\mathrm{HT}_{\mathrm{LB}}$, this set of rules has a single solution assigning 2000 to rpm and 200 to range. The foundedness prunes other Pareto minimal assignments like the one mapping 4000 to rpm and 100 to range, since there is no proof of $rpm \geq 4000$.

An alternative way to formalize semantics of logic programs is by means of fixpoints of a semantic operator. This line of research was initiated by Fitting [8] and later further developed into a highly generic (algebraic) theory called *Approximation Fixpoint Theory* (AFT) [6].

AFT has proven to be very successful for formalizing semantics of extensions of logic programs (e.g. with aggregates [11]). Essentially, to apply AFT to extensions of LP only a three-valued truth evaluation of rule bodies is needed. Since a fragment of $\mathrm{HT}_{\mathrm{LB}}$ constitutes an extension of normal programs, the question raises whether their semantics can also be captured easily by AFT.

When defining stable fixpoints, AFT essentially minimizes in function of the given lattice. Thus, we may directly insert the order used in $\mathrm{HT}_{\mathrm{LB}}$ (in case this is a lattice itself) to guarantee foundedness. Moreover, there are many correspondences between HT and AFT, e.g., AFT was originally based on two-tuples of elements, similar to HT interpretations. The well-founded semantics based on HT makes use of tuples of two HT-interpretations as intermediary concepts. This is reminiscent of the two-input single step operator of Fitting [8], that was simplified in AFT; an immediate question here is whether the semantics can be obtained as well by using a simpler operator for HT.

In an upcoming work we investigate a rule-based fragment of $\mathrm{HT}_{\mathrm{LB}}$, similar to normal programs, using AFT. Therefore, we define a semantic operator that immediately allows us to derive a stable and well-founded semantics (and others). There, we show that the $\mathrm{HT}_{\mathrm{LB}}$ semantics and the AFT-induced semantics coincide if all atoms are similar in nature to *convex* aggregates [1] - otherwise not necessarily. This result is familiar to the situation for aggregates.

In general, our result provides confidence in the $\mathrm{HT}_{\mathrm{LB}}$ semantics, by showing that it corresponds, on an important class of projects to the semantics defined by AFT, we are guaranteed that well-established non-monotonic reasoning principles are respected in this semantics. Additionally, the application of AFT suggests a strong relationship between weighted ADFs (formalized in AFT in [3]) and $\mathrm{HT}_{\mathrm{LB}}$. Our hypothesis is that wADFs relate to $\mathrm{HT}_{\mathrm{LB}}$ in a similar way as to how *ultimate* semantics coincide with standard semantics for logic programming [7], having higher precision at a higher computational cost. Furthermore, for that class of programs, we now immediately get access to a rich theory, including algebraic stratification results [5] and predicate introduction results [12] that can be translated back into $\mathrm{HT}_{\mathrm{LB}}$. This also further motivates our goal to extend these ideas to general $\mathrm{HT}_{\mathrm{LB}}$ (or at least, to a broader subclass of it).

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